Eur. Phys. J. A **14**, 169–178 (2002)
DOI 10.1140/epja/i2002-10004-2

THE EUROPEAN PHYSICAL JOURNAL A

Langevin description of charge fluctuations in fission of highly excited nuclei

A.V. Karpov[a] and G.D. Adeev[b]

Department of Theoretical Physics, Omsk State University, Mira prospect 55-A, Omsk, 644077 Russia

Received: 22 October 2001 / Revised version: 25 March 2002
Communicated by J. Äystö

Abstract. A stochastic approach to fission dynamics based on three-dimensional Langevin equations is used to study charge fluctuations in fission of highly excited nuclei. Elongation, neck and charge asymmetry parameters are chosen as relevant collective coordinates. Using as an example ^{4}He $+^{232}$ Th induced fission of ^{236}U in a broad range of excitation energies (from 60 to 160 MeV), the isobaric charge variance is investigated to obtain information on nuclear dissipation. The friction parameter of the charge mode is calculated under the assumption of both the one-body and two-body mechanisms of nuclear viscosity. The results obtained for the variances of the charge distribution within the applied approach reveal that the optimal reproduction of the available data is achieved with the value of the two-body viscosity coefficient $(0.6 \leq \nu_0 \leq 1.8) \times 10^{-23}$MeV s fm^{-3} that is close to those deduced earlier from the description of the fission fragment mass-energy distribution. The expression for the friction parameter of the charge mode is derived within the one-body mechanism of nuclear dissipation. The one-body mechanism of nuclear dissipation also provides a good reproduction of data on the isobaric charge variance without any adjustable strength parameter.

PACS. 25.85.-w Fission reactions – 05.10.Gg Stochastic analysis methods (Fokker-Planck, Langevin, etc.) – 02.50.Ey Stochastic processes

1 Introduction

In the last decade the fission of highly excited nuclei has been studied from the viewpoint of dissipative large-scale collective nuclear motion at a finite temperature. The understanding of the role of nuclear dissipation for such processes as deep-inelastic heavy-ion collisions [1], damping of giant resonances [2], and induced fission [3] is one of the most spectacular achievements of contemporary nuclear physics. Nevertheless, a common conclusion on the dissipation strength or the friction coefficient for collective motion of nuclear matter at a finite temperature is not reached yet. Presently, there is no unambiguous information about the deformation and/or the temperature dependence of nuclear dissipation. Information about the dissipation strength for collective motion of nuclear matter at a finite temperature is traditionally deduced from the analysis of experimental data on multiplicities of prescission light particles and photons, on fission fragment mass-energy distributions from excited compound nuclei, and on the fission probability (cross-sections for evaporation residue formation). It is symbolic that compilations of the available estimates for the friction coefficient obtained both from more or less *a priori* theories and from the analysis of the above-mentioned experimental data are presented in ref. [4] in logarithmic scale. The elucidation of the mechanism of nuclear viscosity and the reliable estimation of its value continue being an essentially open question. Therefore, it is desirable to extend the number of observables, whose analysis can provide additional information about the dissipation strength of collective motion of nuclear matter. We deem that charge distributions [5] as well as angular distributions [6,7] of fragments originating from the fission of heavy nuclei can be a valuable source of information about the dynamics of the process. In ref. [6] it has been shown that distinguishing between presaddle and saddle-scission prescission neutrons removes a discrepancy between data for the fission fragment angular anisotropy and an analysis within the conventional transition state theory. From the analysis developed in [6] it is possible to deduce the reduced friction parameter of the main fission mode (the elongation collective coordinate).

In the present paper to study charge fluctuations the main attention has been focused on the calculation of the isobaric charge variance. We have applied the method proposed recently [8] for the description of the fission fragment mass-energy distribution. But the charge asymmetry collective coordinate has been used instead of the mass

[a] e-mail: karpov@phys.omsu.omskreg.ru
[b] e-mail: adeev@univer.omsk.su

asymmetry one. The isobaric charge variance of fission fragments of excited nuclei has been investigated under the assumption of two viscosity mechanisms: the one-body and two-body ones. The research has been carried out by using the three-dimensional Langevin equations for the reaction $^4\mathrm{He} + {}^{232}\mathrm{Th} \to {}^{236}\mathrm{U}$ in a wide range of excitation energies of the compound nucleus $E^* = 60$–160 MeV. The calculations reported in this paper are aimed at drawing a conclusion about the friction with respect to the charge mode and, in particular, at comparing the predictions of the model for both viscosity mechanisms.

2 The model

The well-known $\{c, h, \alpha\}$ parametrization [9] has been used to describe the shape of the fissioning nucleus. Actually, we consider the case of symmetric fission, *i.e.*, the case of $\alpha = 0$. The model can be easily extended to $\alpha \neq 0$. However, consideration of the asymmetric nuclear fission is of minor significance from the viewpoint of the main purpose of the paper (analysis of charge fluctuations) and causes appreciable calculation difficulties. Therefore, we have restricted ourselves to the situation of symmetric fission.

In the stochastic approach [10] the evolution of collective coordinates is treated as a motion of Brownian particles in a "heat bath" constituted by internal degrees of freedom. The system of coupled Langevin equations for three collective coordinates has the form

$$\begin{aligned} \dot{q}_i &= \mu_{ij} p_j, \\ \dot{p}_i &= -\frac{1}{2} p_j p_k \frac{\partial \mu_{jk}}{\partial q_i} - \frac{\partial F}{\partial q_i} - \gamma_{ij}\mu_{jk} p_k + \theta_{ij}\xi_j, \end{aligned} \tag{1}$$

where $i, j, k = 1, 2, 3$; $\mathbf{q} = (c, h, \eta_Z)$ are the collective coordinates; $\mathbf{p} = (p_c, p_h, p_{\eta_Z})$ are the conjugate momenta. The coordinates c and h describe the nuclear shape and the coordinate $\eta_Z = (Z_\mathrm{R} - Z_\mathrm{L})/(Z_\mathrm{R} + Z_\mathrm{L})$, where Z_R and Z_L are charges of the right and left fragments, determines the ratio of the numbers of protons in the right and left fragments. Here and in what follows the subscripts "R" and "L" refer to the right and left fragments, respectively. In (1) m_{ij} ($\|\mu_{ij}\| = \|m_{ij}\|^{-1}$) is the tensor of inertia; γ_{ij} is the friction tensor; $\theta_{ij}\xi_j$ is a random force; and ξ_i is a random variable satisfying the relations $\langle \xi_i \rangle = 0$ and $\langle \xi_i(t_1)\xi_j(t_2) \rangle = 2\delta_{ij}\delta(t_1 - t_2)$, where the angular brackets denote averaging over an ensemble.

We have used the free energy $F(\mathbf{q})$ to calculate the conservative part of the driving force. In the Fermi-gas model the free-energy functional is defined as

$$F(\mathbf{q}) = V(\mathbf{q}) - a(\mathbf{q})T^2, \tag{2}$$

where $V(\mathbf{q})$ is the potential energy, and the level density parameter $a(\mathbf{q})$ is determined as follows:

$$a(\mathbf{q}) = \alpha A + \beta A^{2/3} B_\mathrm{s}(\mathbf{q}). \tag{3}$$

Here A is the mass number of the compound nucleus, B_s is the dimensionless functional of the surface energy, $\alpha = 0.073$ MeV^{-1}, and $\beta = 0.095$ MeV^{-1} [11]. The strengths of the random force θ_{ij} are evaluated from the equation $\theta_{ik}\theta_{kj} = T\gamma_{ij}$. The "heat bath" temperature is given by the Fermi-gas model expression $T = \sqrt{E_\mathrm{int}/a(\mathbf{q})}$, where E_int is the internal excitation energy. Since the charge mode is a finite mode, for determination of $\theta_{\eta_Z\eta_Z}$ one should use an effective temperature T_Z^* [12], which takes into account quantum fluctuations, instead of the "heat-bath" temperature T:

$$T_Z^* = \frac{\hbar\omega_Z}{2} \coth\left(\frac{\hbar\omega_Z}{2T}\right), \tag{4}$$

where ω_Z is the frequency with respect to η_Z.

The Langevin trajectories are simulated starting from the ground state of the compound nucleus. The initial conditions were chosen by the Neumann method with the generating function

$$\begin{aligned} P(\mathbf{q}_0, \mathbf{p}_0, l, t = 0) \sim \exp\left(-\frac{V(\mathbf{q}_0) + \frac{1}{2}\mu_{ij}(\mathbf{q}_0)p_i^0 p_j^0}{T}\right) \\ \times \delta(\mathbf{q} - \mathbf{q}_0)F(l), \end{aligned} \tag{5}$$

where $\mathbf{q}_0 = (c_0 = 1, h_0 = 0, \eta_{Z0} = 0)$ are the coordinates of the ground state. The initial spin distribution function $F(l)$ was taken from ref. [13]. The initial state is assumed to be characterized by the thermal-equilibrium momentum distribution in the coordinates (c, h), the initial value of the momentum $p^0_{\eta_Z} = 0$.

We suppose that if the nuclear shape is so compact that it does not have a well-defined neck it is meaningless to speak about charge division between the fragments. Therefore, evolution of the system with respect to the charge coordinate starts only after appearance of the neck in the shape of the fissioning nucleus, and we shall imply (in discussion of calculations of the potential energy, the transport coefficients of the charge mode, and results concerning dynamics of charge fluctuations) that the neck in the nuclear shape already exists. Nevertheless, the coordinates of the ground state of the compound nucleus were chosen as initial values of the collective coordinates in order to take into account evaporation of the prescission neutrons correctly. As shown in ref. [14], an appreciable part of the prescission neutrons (more than half) is evaporated from the nearly spherical compound nucleus at the early stage of the fission process before the saddle point is reached. Our dynamical calculations give a linear dependence of the mean prescission neutron multiplicity on the excitation energy. The multiplicity varies from 1.32 for $E^* = 60$ MeV to 5.96 for $E^* = 160$ MeV. It is well known that each evaporated neutron carries away about 8–10 MeV of the total excitation energy. It is obvious that dynamical calculations, if carried out without evaporation of the prescission neutrons, lead to a considerable overestimation of the nuclear excitation energy on the descent from the saddle to scission and, hence, to an inadequate description of fission fragment distributions.

The choice of the initial conditions in the form of eq. (5) means that we start the Langevin calculations

from the stage of thermally equilibrated excited nucleus. The concept of an equilibrated compound nucleus with fixed excitation energy appears to be a rather unrealistic idealization of the real complicated situation. In any case the reaction consists of a fast preequilibrium stage and slower decay of the remaining thermally equilibrated excited compound nucleus. The equilibration phase can be described in a proper way in the internuclear cascade model [15]. When the cascade stage is over, a thermalized residual nucleus is formed. Due to the fluctuations in the cascade shower, the thermalized residuals have wide distributions in their overall characteristics: their proton and neutron numbers, excitation energy, linear and angular momenta. The internuclear cascade model is the only model which allows to account for these fluctuations and to determine the whole distribution of the parameters characterizing the compound nucleus. The consideration of the preequilibrium stage of the reaction is necessary for comparison of the calculated results with experimental data at high excitation energy $E^* > (150–200)$ MeV. At lower excitations $E^* < 100$ MeV the concept of statistically equilibrated nucleus is a reasonable approximation of the initial conditions in the modeling of the fission dynamics.

Let us introduce the notion of a mean trajectory that will be frequently used in the paper. The mean trajectory is a trajectory obtained in our dynamical Langevin calculations by averaging over a trajectory ensemble. In this case the Langevin equations are reduced to the generalized Hamilton equations, since the term responsible for fluctuations (the random force) drops out after averaging (see [23]). The neck radius being equal to $0.3R_0$, where R_0 is the radius of an initial spherical nucleus [9,8], was chosen as a scission condition. This condition determines the scission surface in the space of the collective coordinates. Intersection of stochastic trajectories with the scission surface forms an ensemble of scission points. Averaging over this ensemble gives the mean scission point. The mean scission point determines average characteristics of observables (for example, the mean charge or mean kinetic energy of the fission fragments). Spread of the scission points relative to the mean scission point determines variances of the observables (for example, widths of the charge or kinetic-energy distributions of the fission fragments).

The liquid-drop model (LDM) with the Myers-Swiatecki coefficients [16] has been used to calculate the potential energy. The potential energy was obtained as a sum of the Coulomb energy, the symmetry energy, and the rotational energy, as usual. In order to calculate the charge division between the nascent fragments, one has to know the charge density distribution in the fissioning nucleus. Ideally this should be determined from the solution of a variational problem in the framework of a macroscopic model, either the LDM or the droplet model. Alternatively, a simpler method we have used in this work is to introduce a new collective coordinate related to the charge density distribution. Such collective coordinate (η_Z) has been mentioned above. We have used the simplest assumption about the charge density distribution, namely the charge density is different in the nascent fragments, but it is constant inside each of them [17]:

$$\rho^{\mathrm{p}}(\mathbf{r}) = \begin{cases} \rho^{\mathrm{p}}_{\mathrm{R}}, & \mathbf{r} \in V_{\mathrm{R}}\,, \\ \rho^{\mathrm{p}}_{\mathrm{L}}, & \mathbf{r} \in V_{\mathrm{L}}\,, \end{cases} \tag{6}$$

where V_{R} and V_{L} are the volumes of the fragments. The total density is assumed to be constant throughout the volume of the fissioning nucleus:

$$\rho^{\mathrm{p}}_{\mathrm{R}} + \rho^{\mathrm{n}}_{\mathrm{R}} = \rho^{\mathrm{p}}_{\mathrm{L}} + \rho^{\mathrm{n}}_{\mathrm{L}} = \rho^{\mathrm{p}}_0 + \rho^{\mathrm{n}}_0 = \rho, \tag{7}$$

where $\rho^{\mathrm{p}}_0 = Z/(4/3\pi R_0^3)$, $\rho^{\mathrm{n}}_0 = N/(4/3\pi R_0^3)$, and ρ are the proton, neutron and total densities for the uniform charge distribution in the nucleus. The charge densities $\rho^{\mathrm{p}}_{\mathrm{R}}$ and $\rho^{\mathrm{p}}_{\mathrm{L}}$ can be expressed in terms of the parameter η_Z and the charge density ρ^{p}_0 of the original nucleus by

$$\rho^{\mathrm{p}}_{\mathrm{R}} = \rho^{\mathrm{p}}_0 \frac{(k+1)}{2k}(1+\eta_Z)\,, \quad \rho^{\mathrm{p}}_{\mathrm{L}} = \rho^{\mathrm{p}}_0 \frac{(k+1)}{2}(1-\eta_Z). \tag{8}$$

Here $k = A_{\mathrm{R}}/A_{\mathrm{L}}$ is the mass ratio of the nascent fragments. The expressions for the neutron densities $\rho^{\mathrm{n}}_{\mathrm{R}}$ and $\rho^{\mathrm{n}}_{\mathrm{L}}$ can be obtained from eqs. (7) and (8). It should be noted that the model proposed in ref. [17] allows to describe independent division of the mass and charge during fission not only for the separated fragments but also for all continuous shapes of the fissioning nucleus directly prior to scission. Therefore, the model for independent division of the charge and mass for shapes with a pronounced neck allows a natural treatment of the formation of the fragment charge distribution in dynamical approaches including the stochastic one. Despite its simplicity, the model was found to work well in the description of the charge and mass distributions in fission [17,18] and of the products of reactions with heavy ions [19].

The symmetry energy makes the dominant contribution to the potential energy as a function of the collective coordinate η_Z. For an arbitrary nuclear shape it can be calculated under the assumption that this energy is uniformly distributed over the whole nucleus and determined by the integral

$$V_{\mathrm{sym}} = a_{\mathrm{sym}} \int \frac{\left[\rho^{\mathrm{n}}(\mathbf{r}) - \rho^{\mathrm{p}}(\mathbf{r})\right]^2}{\rho} \mathrm{d}V, \tag{9}$$

where $a_{\mathrm{sym}} = 23.7$ MeV is the coefficient of the symmetry energy. The expressions for the Coulomb and symmetry energies for the chosen charge density distribution are given in refs. [17,18,20]. The potential liquid-drop energy as a function of the parameter η_Z is described by an oscillator dependence which is characterized by a stiffness coefficient with respect to variations of the coordinate η_Z

$$C_{\eta_Z} = 2\left(\frac{(k+1)Z}{kA}\right)^2 \times \left(a_{\mathrm{sym}} kA + \frac{E^0_{\mathrm{C}}}{(1-\delta)^2}\left[(1+k)\left(B^{\mathrm{R}}_{\mathrm{C}} + kB^{\mathrm{L}}_{\mathrm{C}}\right) - kB_{\mathrm{C}}\right]\right), \tag{10}$$

where $\delta = \frac{N-Z}{A}$ is the neutron excess per nucleon in the initial nucleus; $B_{\rm C}$, $B_{\rm C}^{\rm R}$, and $B_{\rm C}^{\rm L}$ are the Coulomb energies of the whole nucleus and the nascent fragments with original charge density in units of the Coulomb energy of the original spherical nucleus $E_{\rm C}^0$. The stiffness coefficient C_{η_Z} is not very sensitive to the geometrical configuration of the fissioning system at scission. This justifies the estimates usually made for the stiffness C_{η_Z} [21] for a simplified scission configuration in the form of spherical fragments. The values of C_{η_Z} given by (10) are equal to $C_{\eta_Z} = (7\text{–}8) \times 10^3$ MeV. The stiffness C_{η_Z} does not change strongly, and during the descent of the fissioning system from the saddle point to scission it increases no more than 20%.

Reliable determination of the transport coefficients entering the Langevin equations (1) remains one of the principal unresolved problems of the collective nuclear dynamics (see reviews [1,22] and references therein). In our case the situation becomes even more complicated since two of the collective coordinates (c, h) are the shape parameters, while the third one (η_Z) is related to the charge redistribution during the fission process and has quite another physical nature. There are well-developed approaches to the calculation of the transport coefficients connected with the shape collective coordinates [23]. On the other hand, there is no well-elaborated approach to computing the transport coefficients of the charge mode. So the charge mode transport coefficients (inertia parameter $m_{\eta_Z\eta_Z}$ and friction parameter $\gamma_{\eta_Z\eta_Z}$) are often assumed to be coordinate-independent free variable parameters [18,24], or very simple hydrodynamical models [21,25] are used to estimate these transport coefficients of the charge mode. In the present work the components of the inertia tensor m_{cc}, m_{ch}, and m_{hh} were calculated by means of the Werner-Wheeler method [23]. A modified one-body mechanism of nuclear dissipation [1,2,26] with the reduction coefficient $k_s = 0.25$ [2,8] has been used for the determination of the friction tensor components γ_{cc}, γ_{ch}, and γ_{hh}. Nondiagonal elements of the inertia tensor $m_{c\eta_Z}$ and $m_{h\eta_Z}$ and the respective components of the friction tensor $\gamma_{c\eta_Z}$ and $\gamma_{h\eta_Z}$ were supposed to be zero.

The frequently used expression [27] for calculation of the mass parameter of the charge mode is

$$m_{\eta_Z\eta_Z}(c,h) = \frac{\pi}{6} r_0^3 m \frac{ZA^2}{N} \frac{1}{r_N}. \tag{11}$$

This expression is not suitable for the problem discussed by two reasons. First, it is obtained for a flow of an inviscid liquid, whereas viscosity plays an essential role in our model. Second, for deformations close to scission the nascent fragments are connected by a rather long neck that is completely ignored in (11) (since the nuclear shape is approximated by two spheres joined together by a circular hole of radius r_N). We have used the expression obtained in ref. [21] for a flow of viscous incompressible liquid through a cylindrical neck connecting two spherical parts for the determination of the inertia parameter of the charge mode

$$m_{\eta_Z\eta_Z}(c,h) = \frac{1}{3\pi} \frac{mZA^2}{\rho N} \frac{l + 2r_N}{r_N^2}, \tag{12}$$

where l is the neck length. Figure 1 shows the shape of the nucleus at the mean scission point (thin curve) and the way it is approximated to be used in eq. (12) (thick curve). The centers of the spheres coincide with the mass centers of the fragments. The neck length l is determined from the condition of the volume conservation of the nucleus. The nuclear shape is fitted in the same way (see below) to calculate the friction parameter of the charge mode with the two-body viscosity mechanism.

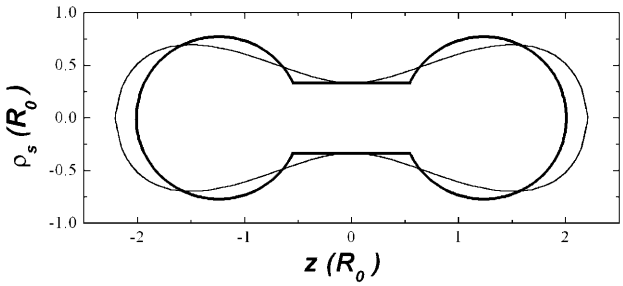

Fig. 1. The thin curve is the nuclear shape in $\{c, h, \alpha\}$ parametrization at the mean scission point; the thick curve is the approximation of this shape for calculation of the mass parameter of the charge mode and the friction parameter under the assumption of the two-body viscosity mechanism.

As mentioned above, it is a crucial problem how to take into account the influence of the nuclear deformation on the friction parameter of the charge mode. In the present work the coordinate dependence of the parameter $\gamma_{\eta_Z\eta_Z}$ was taken into account in two ways —under the assumption of the one-body and two-body viscosity mechanisms. The simplest method to evaluate the friction parameter is to apply the hydrodynamical model. In ref. [21] the dissipation of the isovector vibrations was estimated by means of a model of steady flow of a viscous incompressible liquid along a cylindrical tube of length l and of radius r_N, which plays the role of a neck connecting the nascent fragments for the considered model. In this case the field of velocities obtained from the solution of the Navier-Stokes equation has the only component along the symmetry axis of the nucleus z and depends on the cylindrical coordinate r only [25]:

$$u(r) = u_z(r) = \frac{\Delta p}{4\nu_0 l} \left(r_N^2 - r^2\right), \tag{13}$$

where Δp is the pressure difference across the ends of the tube, ν_0 is the coefficient of dynamical viscosity. In the general case of an incompressible liquid the kinetic energy of the liquid and the rate of the energy dissipation are given by

$$E_{\rm kin} = \frac{m\rho}{2} \int u^2 {\rm d}V\,,$$
$$\dot{E}_{\rm kin} = -\frac{\nu_0}{2} \int \left(\frac{\partial u_i}{\partial x_k} - \frac{\partial u_k}{\partial x_i}\right)^2 {\rm d}V\,. \tag{14}$$

In (14) the integration over the tube volume is assumed, and i, k are the Cartesian indices. Thus, for the field of velocities (13) one has

$$E_{\rm kin} = \frac{\pi m\rho\,(\Delta p)^2\, r_N^6}{96\nu_0^2 l}\,,$$
$$\dot{E}_{\rm kin} = -\frac{\pi\,(\Delta p)^2\, r_N^4}{8\nu_0 l}\,. \qquad (15)$$

Therefore, the reduced friction parameter of the charge mode is

$$\beta_{\eta_Z\eta_Z} = \frac{\gamma_{\eta_Z\eta_Z}}{m_{\eta_Z\eta_Z}} = -\frac{\dot{E}_{\rm kin}}{2E_{\rm kin}} = \frac{6\nu}{r_N^2}, \qquad (16)$$

where $\nu = \nu_0/(m\rho)$ is the coefficient of kinematic viscosity. The magnitude $\nu_0 = 1.8\times10^{-23}$ MeV s fm^{-3} has been derived in ref. [28] from the experimental data on the widths of the giant dipole resonances (GDR). We have used this value of ν_0 as a starting value of the two-body viscosity coefficient in our dynamical Langevin calculations, because the dipole isovector oscillations along the symmetry axis of the nucleus are suggested to be the main cause of the charge redistribution between the nascent fragments.

A good description of experimental data on the widths of the GDR has been achieved both with the two-body [29] and the one-body viscosity mechanisms [30,31]. Moreover, the one-body viscosity mechanism is more appropriate for Fermi particle systems than the two-body one. Therefore, it is also interesting to apply the one-body dissipation model to calculate the friction parameter of the charge degrees of freedom. It is more convenient for our purposes to write a complete formula expressing the rate of the collective kinetic-energy dissipation in the one-body dissipation model [1,26] as follows:

$$-\left(\frac{{\rm d}E_{\rm kin}}{{\rm d}t}\right) = -\left(\frac{{\rm d}E_{\rm kin}}{{\rm d}t}\right)_{\rm wall} - \left(\frac{{\rm d}E_{\rm kin}}{{\rm d}t}\right)_{\rm window} + m\bar{v}\frac{16}{9}\frac{1}{\rho\Delta\sigma}\dot{N}_1^2\,. \qquad (17)$$

Here $\bar{v}$ is the average particle speed; m is the mass of the particle; $\Delta\sigma$ is the area of the window between two parts of the system; N_1 is the number of particles in one of the nascent fragments (for example, in the right one). The explicit expressions for the first two terms in eq. (17) can be found in refs. [32,8]. The wall formula has been adapted to study the GDR in ref. [31]. The formula for the friction parameter of the charge mode in the one-body dissipation model can be derived in the following way. Equation (17) should be applied separately to the proton and neutron liquids, and the resulting rate of the energy dissipation presents a sum of contributions from the proton and neutron subsystems, *i.e.,* $\dot{E}_{\rm kin} = \dot{E}^{\rm p}_{\rm kin} + \dot{E}^{\rm n}_{\rm kin}$. The condition of the nucleons number conservation in each fragment has to be imposed here. Moreover, the shape parameters must be fixed, because we calculate the rate of the collective kinetic energy dissipation connected with the charge transfer through the window but not with the variation of the nuclear shape. By virtue of these constraints the contribution from the first two terms in (17) vanishes. The last term in (17) was obtained in refs. [1,26] as an additional term to the original wall-and-window formula [33]. This term is associated with the resistance of the system against the change of the mass asymmetry. In the case of the charge degree of freedom this term describes the energy dissipation connected with the relative changing in the number of protons in the fragments without any changes in nuclear shape. Thus, we have obtained the following expression for the friction parameter of the charge mode in the one-body dissipation model:

$$\gamma_{\eta_Z\eta_Z} = \frac{4m}{9\rho}\frac{AZ}{N}\left[N\bar{v}_{\rm p} + Z\bar{v}_{\rm n}\right]\frac{1}{\Delta\sigma}, \qquad (18)$$

where $\bar{v}_{\rm p}$ and $\bar{v}_{\rm n}$ are the proton and neutron average velocities inside the nucleus, respectively. Figure 2 shows the dependence of the friction parameter of the charge mode on the elongation parameter c under the assumption of the two-body (continuous curve) and the one-body (dashed curve) viscosity mechanisms. It should be noted that the deformation dependence of $\gamma_{\eta_Z\eta_Z}$ in both viscosity mechanisms is determined by the neck thickness only. One has $\gamma_{\eta_Z\eta_Z} \sim 1/r_N^2$ for the one-body viscosity mechanism and $\gamma_{\eta_Z\eta_Z} \sim 1/r_N^4$ for the two-body one. It can be seen from fig. 2 that both viscosities give close values of the friction parameter in a wide range of nuclear deformations. But when the system is approaching scission the friction parameter $\gamma_{\eta_Z\eta_Z}$ calculated with the two-body viscosity mechanism increases extremely sharp and at the mean scission point it exceeds prediction of the one-body dissipation model more than twice. One more important distinction between two viscosity mechanisms is the presence of the adjustable coefficient ν_0 in the hydrodynamical model of two-body viscosity and absence of any variable coefficients in the one-body dissipation model.

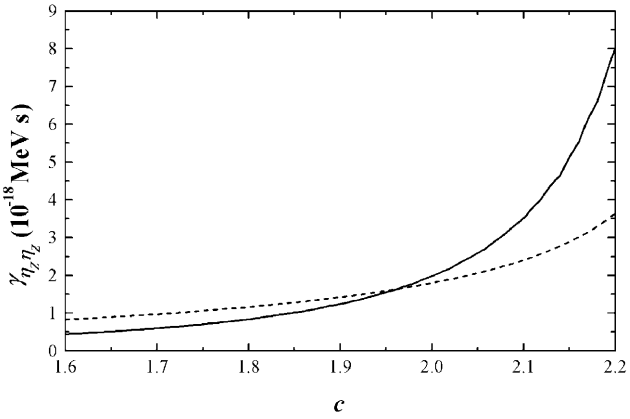

Fig. 2. The dependence of the friction parameter of the charge mode on the elongation parameter c along the mean trajectory. The continuous curve is the calculation with the two-body viscosity mechanism ($\nu_0 = 1.8\times10^{-23}$ MeV s fm^{-3}) and the dashed curve is obtained with the one-body viscosity.

The problem of temperature dependence of the transport coefficients in the Langevin equations (1) (in par-

ticular, of the friction parameters) is still open [34]. Unfortunately, quite controversial information is reflected in literature. The temperature dependence of nuclear dissipation is predicted by different theories to decrease as $1/T^2$ [35], to be constant, or to increase with temperature [36]. Moreover, we suppose that the incorporation of the temperature-dependent friction parameters does not alter the main conclusions of the paper. For these reasons we have not taken into account any temperature dependence of $\gamma_{\eta_Z\eta_Z}$.

3 Results and discussions

It is well known that the charge distribution has the form of a curve with one maximum and is usually approximated in both the experimental and the theoretical studies by a Gaussian function characterized by the mean value $\langle Z\rangle$ and the variance σ_Z^2 (symmetric fission leads to $\langle Z\rangle = Z/2$). Earlier the isobaric charge distribution for the fission of the compound nucleus ^{236}U has been investigated theoretically on the basis of the multidimensional Fokker-Planck equation in refs. [18,24] but with the coordinate-independent friction parameter. Also there are a lot of experimental data on the variance of the isobaric charge distributions for the fission of the compound nucleus ^{236}U at low excitation energies. In particular, it is known [5, 37] that for the thermal-neutron fission of ^{235}U the charge variance $\sigma_Z^2 = 0.4 \pm 0.05$ and does not depend on the excitation energy. Such behavior of the variance indicates a quantum character of formation of the isobaric charge distributions at low energies. Also in this energy region the experimental data on σ_Z^2 are well described by the expression of a statistical limit $\sigma_{Z,\mathrm{st}}^2 = T_Z^*(\langle \mathbf{q}_{\mathrm{sc}}\rangle)/C_Z(\langle \mathbf{q}_{\mathrm{sc}}\rangle)$, where $C_Z(\langle \mathbf{q}_{\mathrm{sc}}\rangle) = 4C_{\eta_Z}(\langle \mathbf{q}_{\mathrm{sc}}\rangle)/Z^2$ and $\langle \mathbf{q}_{\mathrm{sc}}\rangle$ are coordinates of the mean scission point. If $T \ll \hbar\omega_Z/2$ than $T_Z^* \simeq \hbar\omega_Z/2$ that explains the above-mentioned constancy of the charge variance for the low-energy fission.

Our dynamical model cannot be applied to the description of the low-energy fission, because shell effects and effects of nucleons pairing are not taken into account in the calculation of the potential energy and transport coefficients. However, we can calculate the magnitude $\sigma_{Z,\mathrm{st}}^2$ and compare it with the experimental data. For the thermal-neutron–induced fission of the uranium nucleus ($E^* = 6.4$ MeV) we have obtained $\sigma_{Z,\mathrm{st}}^2 = 0.35$, that is in accordance with the experimental value of the charge variance.

3.1 The relaxation times of the charge mode

Let us turn now to the results of the dynamical calculations. First of all, we shall discuss characteristic times of the charge mode. For a system with dissipation, it is natural to use the relaxation time as a characteristic time [38]

$$\tau_{\eta_Z} = \begin{cases} 2\tilde{\beta}_{\eta_Z}^{-1}, & \omega_{\eta_Z} \geq \tilde{\beta}_{\eta_Z}/2 \\ \left[\tilde{\beta}_{\eta_Z}/2 - \left(\tilde{\beta}_{\eta_Z}^2/4 - \omega_{\eta_Z}^2\right)^{1/2}\right]^{-1}, & \omega_{\eta_Z} < \tilde{\beta}_{\eta_Z}/2, \end{cases} \tag{19}$$

where $\tilde{\beta}_{\eta_Z} = \beta_{\eta_Z} + \dot{m}_{\eta_Z\eta_Z}/m_{\eta_Z\eta_Z}$ is the generalized damping coefficient of the charge mode. In (19) the first case ($\omega_{\eta_Z} \geq \tilde{\beta}_{\eta_Z}/2$) corresponds to the regime of damping oscillations (underdamped motion) and the second case $\omega_{\eta_Z} < \tilde{\beta}_{\eta_Z}/2$ to the regime of aperiodic damping (overdamped motion). The calculated results for τ_{η_Z} are shown in fig. 3 for both viscosity mechanisms. One can see that the coordinate dependence of the relaxation time is strongly influenced by the two-body viscosity coefficient. In particular, τ_{η_Z} decreases with increasing nuclear deformation when the charge oscillator is underdamped (figs. 3(a),(b)), and, on the contrary, τ_{η_Z} increases when the charge oscillator is overdamped (figs. 3(e),(f)). The situations shown in figs. 3(c),(d) correspond to the regime of damping oscillations in the beginning of the evolution of the charge mode but at $c \simeq 2.1$ for $\nu_0 = 1.8 \times 10^{-23}$ MeV s fm^{-3} and at $c \simeq 1.75$ for $\nu_0 = 5.7 \times 10^{-23}$ MeV s fm^{-3} the system passes into the regime of aperiodic damping (the kink in the figures). Then we shall add some remarks on the behaviour of τ_{η_Z} in the one-body dissipation model. Figure 3(c) indicates that under the assumption of the one-body viscosity mechanism the system is in the regime of damping oscillations over the entire descent of the nucleus from the saddle point to scission. Besides, the one-body viscosity mechanism, in contrast to the two-body one, gives approximately a constant value of the relaxation time $\tau_{\eta_Z} \simeq 0.4 \times 10^{-21}$ s. It can be easily understood from eqs. (12), (18), and (19) having taken into consideration that the neck length does not practically depend on the nuclear deformation, and the ratio $\dot{m}_{\eta_Z\eta_Z}/m_{\eta_Z\eta_Z}$ is minor in comparison with β_{η_Z}.

One more important question arisen in connection with the discussion of the relaxation times of the charge mode is an applicability of the Langevin equations for the description of charge fluctuations. We have used the Langevin equations in the Markovian approximation. It suggests that relaxation time of intrinsic degrees of freedom τ_{int} is much smaller in comparison with that of a considered collective mode. It is shown in ref. [39] that τ_{int} is about 0.2×10^{-21} s. We suppose that the Markov limit is justified if the relaxation time of the charge degree of freedom is at least about two or three times larger than τ_{int} (*i.e.* $\tau_{\eta_Z} > (0.4 - 0.6) \times 10^{-21}$ s). As mentioned above $\tau_{\eta_Z} \simeq 0.4 \times 10^{-21}$ s in the case of the one-body dissipation mechanism that is on the border of applicability of the Markov limit. In the case of the two-body viscosity mechanism τ_{η_Z} increases with increasing the viscosity coefficient ν_0 if charge fluctuations are overdamped and an increase of ν_0 leads to a decrease of τ_{η_Z}, if the charge motion is underdamped. So there is no problem with the validity of the Markovian approximation for $\nu_0 < 0.57 \times 10^{-23}$ MeV s fm^{-3} and $\nu_0 > 1.8 \times 10^{-23}$ MeV s fm^{-3}. The most problematic case (among all considered) is the case of fig. 3(c). The minimal value reached by the charge relaxation time is about 0.3×10^{-21} s and the Markov limit is not justified there. But in the region determining the parameters of the charge distribution (near the scission point) $\tau_{\eta_Z} = (0.7 - 0.9) \times 10^{-21}$ s. This is at least three times larger than τ_{int}. Thus, the above-mentioned argu-

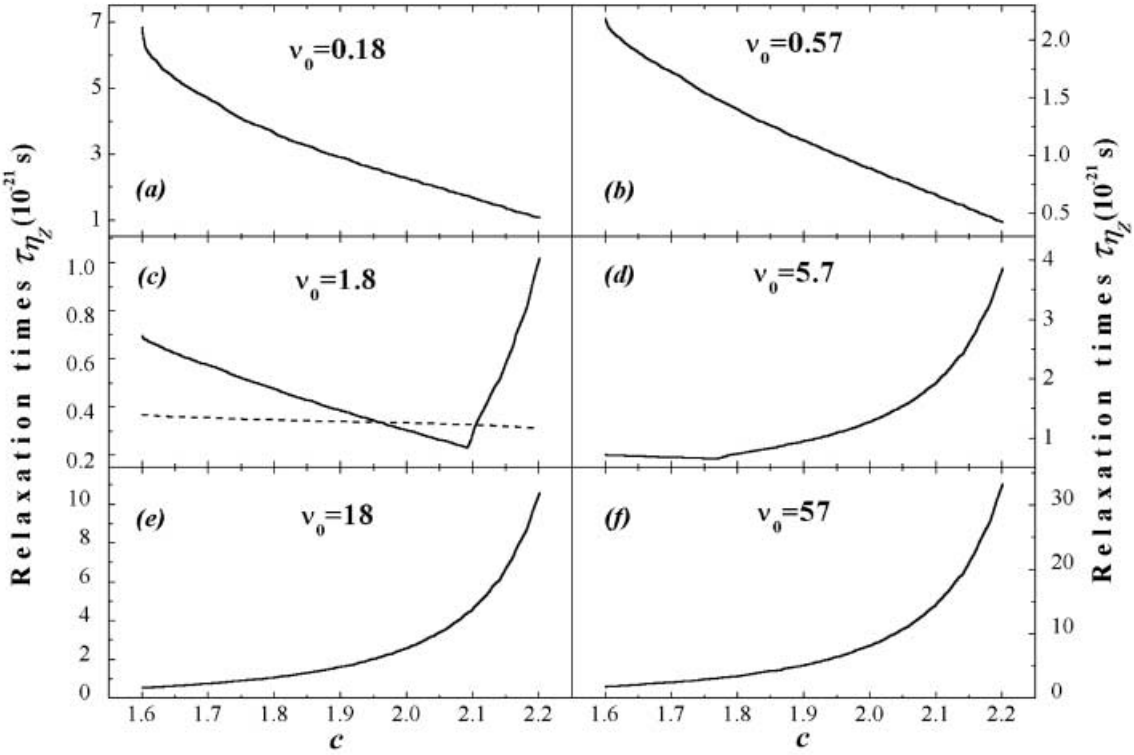

Fig. 3. The relaxation times τ_{η_Z} of the charge mode as functions of the elongation coordinate c along the mean trajectory. Continuous curves are obtained with the two-body viscosity mechanism for a number of the viscosity coefficients. Values of the viscosity coefficient are shown in units ($\times 10^{-23}$ MeV s fm^{-3}). The dashed curve is the calculation under the assumption of the one-body viscosity mechanism.

ments give us a certainty that all the calculated parameters of the charge distribution on the basis of the Langevin equations in the Markovian approximation are correct. In our opinion the use of the Langevin equations in the Markov limit is the first but necessary step in solving the problem. On the other hand, if the Markovian approximation is not valid one can use the memory-dependent Langevin equations with retarded friction [40]. Possible improvements in this direction could be made in future studies.

Figure 3(c) shows that the relaxation times $\tau_{\eta z}$ for both mechanisms of nuclear viscosity do not exceed 10^{-21} s that is much less than the characteristic times of collective modes responsible for the nuclear deformation [38]. Therefore, it can be expected that not only at low but also at high excitation energies the charge mode is equilibrated. In order to verify this statement we have calculated the variance σ_Z^2 as a function of the mean internal excitation energy at scission $E_{\rm int}$. The dependence of the charge variance on $E_{\rm int}$ is shown in fig. 4 in comparison with the curve of the statistical limit. From the figure it is seen that the variance of the charge distribution obtained in our dynamical calculations can be reproduced by the statistical-limit expression quite well. It is also remarkable that the curve of the statistical limit passes through the experimental value of the charge variance at low energies. These facts confirm the supposition about the charge mode equilibration at high excitation energies during the descent of the fissioning nucleus from the saddle to scission.

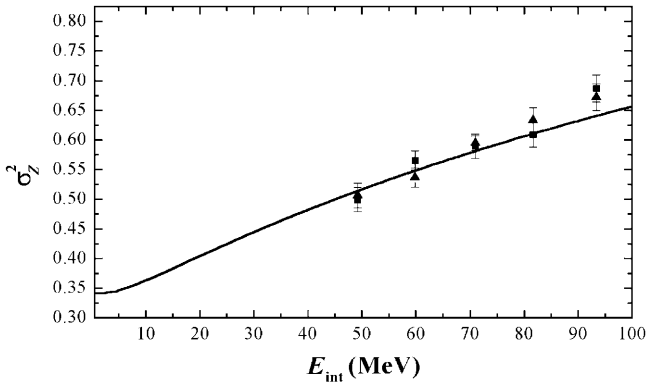

Fig. 4. The variance of the charge distribution σ_Z^2 as a function of the mean internal excitation energy at scission. Squares and triangles are the calculations under the assumption of the two-body ($\nu_0 = 1.8 \times 10^{-23}$ MeV s fm^{-3}) and one-body viscosities, respectively. The continuous line is the statistical limit at the mean scission point.

The second derivation of $\sigma^2_{Z,\rm st}$ on the internal energy calculated at the mean scission point provides additional significant information about the mechanism of formation of the charge distribution. We have come to the following conclusions from the results of our calcu-

lations. The energy axis can be divided into two intervals: $E_{\rm int} < 20$ MeV and $E_{\rm int} > 20$ MeV. In the first interval $\left(\partial^2\sigma^2_{Z,{\rm st}}/\partial E^2_{\rm int}\right) > 0$, it means that the quantum fluctuations play the dominant role as the mechanism of formation of the charge distribution. For the second interval of the internal excitation energy one has $\left(\partial^2\sigma^2_{Z,{\rm st}}/\partial E^2_{\rm int}\right) < 0$, and charge fluctuations in fission have mainly thermal nature. It is noteworthy that in ref. [41] the charge distribution was investigated experimentally for helium-ion–induced fission of ^{232}Th for various excitation energies ranging from 20 to 57 MeV. The charge distribution was found to be Gaussian and independent of the excitation energy (up to 39 MeV). The values of the charge variance that best fit all energies lie in the interval 0.45–0.50. Our calculations are in a good agreement with these results. From fig. 4 it is also clear that both viscosity mechanisms give almost equal charge variances within statistical errors emerging due to a finite number of the Langevin trajectories (in our calculations about 10^4 trajectories). It indicates a weak sensitivity of the width of the isobaric charge distribution to the magnitude of nuclear viscosity.

3.2 The deduced value of the two-body viscosity coefficient

As shown above, at the present time there is no well-developed approach to describe the dissipation of dipole isovector vibrations in fissioning nuclei. The dependence of the friction parameter of the charge mode on the collective coordinates can be described by simple hydrodynamical models (see eqs. (12) and (16)) within the two-body viscosity mechanism. The dissipation strength of the charge mode characterized by the two-body viscosity coefficient ν_0 can be deduced from the confrontation of the data on the charge distribution of fission fragments with the results calculated for the parameters of this distribution, particularly its variance. Bearing in mind this ultimate objective of the present work, we have carried out the calculations of σ^2_Z in a wide range of the two-body viscosity coefficient $\nu_0 = (0.18\text{–}57) \times 10^{-23}$ MeV s fm^{-3}. The results are shown in fig. 5 in logarithmic scale. It can be seen from fig. 5 that the interval $(0.6 \le \nu_0 \le 1.8) \times 10^{-23}$ MeV s fm^{-3} results in a good agreement of the calculated variances σ^2_Z with the value of the statistical limit and, hence (according to the above-mentioned conclusion about charge equilibration), with the experimental data on σ^2_Z. This interval is our evaluation of the two-body viscosity coefficient.

Coordinate dependence and magnitudes of the relaxation times τ_{η_Z} plotted in fig. 3 for all considered viscosity coefficients give us the key to the behavior of the charge variance σ^2_Z. The statistical equilibrium on the charge mode would be established if the relaxation time τ_{η_Z} had been much less than the time of descent $\tau_{\rm ss}$ of the fissioning nucleus from the saddle to scission ($\tau_{\rm ss}$ is about (5–10) $\times 10^{-21}$ s). One can see from fig. 3(b) that in the most valuable (for the parameters of the charge distribution) region of nuclear deformations near

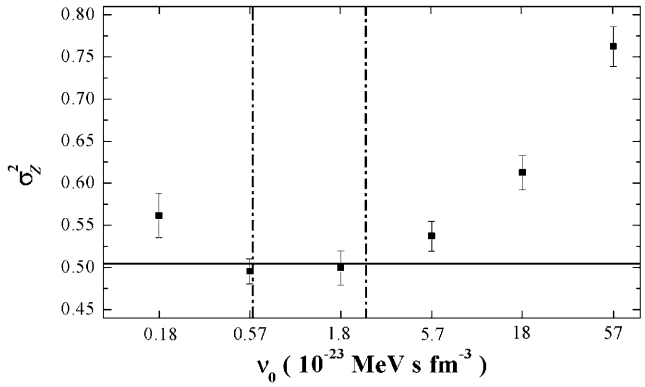

Fig. 5. Dependence of the charge variance σ^2_Z on the viscosity coefficient ν_0. The solid line corresponds to the statistical limit. Vertical dashed lines restrict an interval of values of the two-body viscosity coefficient from refs. [23,29,42,43]. Calculations have been carried out at the excitation energy $E^* = 60$ MeV.

the scission $\tau_{\eta_Z} \simeq 0.5 \times 10^{-21}$ s $\ll \tau_{\rm ss}$. In the case plotted in fig. 3(c) $\tau_{\eta_Z} < 10^{-21}$ s over the entire descent from the saddle to scission. A rapid growth of the relaxation time of the charge mode on the final stage of the descent cannot lead to an appreciable deviation of the dynamically calculated charge variance from the statistical limit. Therefore, we conclude that in the interval $(0.6 \le \nu_0 \le 1.8) \times 10^{-23}$ MeV s fm^{-3} the behavior and values of the relaxation time τ_{η_Z} have to result in validity of the statistical description of the charge variance. On the other hand, for $\nu_0 < 0.57 \times 10^{-23}$ MeV s fm^{-3} (fig. 3(a)) and for $\nu_0 > 1.8 \times 10^{-23}$ MeV s fm^{-3} (figs. 3(d)-(f)) values of the relaxation time τ_{η_Z} are comparable with $\tau_{\rm ss}$ and, hence, the statistical equilibrium at scission is not established. Indeed, one can see deviation of the calculated values of σ^2_Z from the statistical limit.

The mechanism which determines the growth of the charge variance in the region $\nu_0 > 1.8 \times 10^{-23}$ MeV s fm^{-3} is a "memory effect". The essence of this effect can be explained in the following way. Approximate permanence of the stiffness coefficient C_{η_Z} and increase of the mass parameter of the charge mode with increasing deformation leads to the decrease of the effective temperature T^*_Z. It results in decrease of charge fluctuations and in narrowing of the equilibrium charge distribution (determined by the relation T^*_Z/C_Z). On the other hand, rapid growth of the friction parameter $\gamma_{\eta_Z\eta_Z}$ leads to "freezing out" of the charge degree of freedom and σ^2_Z does not change its value during the further descent of the system up to scission. The "freezing" of the charge mode takes place at the large values of the viscosity coefficient $\nu_0 > 1.8 \times 10^{-23}$ MeV s fm^{-3} corresponding to the overdamped motion. Moreover, the larger the coefficient ν_0, the earlier the charge degree of freedom "freezes out" and, hence, the broader the charge distribution of fission fragments becomes. This mechanism explains qualitatively the growth of the charge variance with increasing ν_0.

Earlier the two-body viscosity was widely used in studying the fission fragment mass-energy distribution and certain conclusions about the magnitude of the nu-

clear viscosity coefficient were made. In refs. [23,42] the mean kinetic energy of fission fragments was calculated in a wide range of $Z^2/A^{1/3}$ values. Fitting of the obtained results to experimental data yielded the following values of the coefficient: $\nu_0 = (0.9 \pm 0.3) \times 10^{-23}$ MeV s fm^{-3} [23]; $\nu_0 = (1.9 \pm 0.6) \times 10^{-23}$ MeV s fm^{-3} [42]. In ref. [43] the mass-energy distribution of fission fragments was investigated on the basis of the multidimensional Fokker-Planck equation. It has been shown there that the results of the dynamical calculations are in agreement with experimental data when $\nu_0 = (1.5 \pm 0.5) \times 10^{-23}$ MeV s fm^{-3}. In ref. [29] the various kinds of the collective nuclear motion have been analyzed (in particular, the separation of nucleus into fragments and the giant dipole resonances). The results of this study have allowed the authors to assert that the nuclear viscosity constant is identical for all collective degrees of freedom and has the value $\nu_0 \simeq 10^{-23}$ MeV s fm^{-3}. It is easy to see that the results of our estimations of the viscosity coefficient are in agreement with the previous evaluations of this quantity.

At last, it is useful to note that the dynamical Langevin calculations performed on the basis of the above model make it possible to obtain the distribution in deformations of the fissioning nucleus near the scission region as well as the distribution of fission events in the internal excitation energy. Knowledge of these two distributions of the fissioning nucleus at the instant of scission allows to calculate the multiplicities of postscission light particles by using standard statistical code. The details of such investigations can be found in [44]. The value of the multiplicities of postscission light particles is needed for comparison with fission product data. In fact, in the present work we study charge fluctuations of the primary fission fragments (before their de-excitation).

4 Conclusions

The main results of the work can be summarized as follows.

1. The stochastic approach based on the three-dimensional Langevin equations allows to describe the isobaric charge variance for high-energy fission successfully. Apparently, the model will give good results at low excitation energies after the appropriate completion.
2. In the present work the dynamical calculations of the charge distribution are carried out under the assumption of the one-body and two-body viscosity mechanisms. The friction parameter of the charge mode has different coordinate dependences in the two mechanisms of nuclear viscosity but on an absolute value both viscosity mechanisms give close magnitudes almost up to scission. The important difference of the one-body viscosity mechanism from the two-body one is the absence of any variable coefficients in the one-body dissipation model.
3. Despite serious differences between the viscosity mechanisms they lead to the practically identical values of the variance of the charge distributions (within limits of statistical errors).
4. The calculations of the relaxation times show that not only at low but also at high excitation energies the statistical equilibrium with respect to the charge mode should be established. Such conclusion is confirmed with success by our dynamical calculations of the charge variance as a function of the mean internal excitation energy at scission.
5. The statistical limit is reached with a good accuracy both with the one-body viscosity and the two-body viscosity with the coefficient $(0.6 \leq \nu_0 \leq 1.8) \times 10^{-23}$ MeV s fm^{-3}. The value of the two-body viscosity coefficient found in the present work from the consideration of the dynamical evolution of the charge mode agrees with the former evaluations of this coefficient made for other degrees of freedom of the nucleus [23, 29,42,43].

The authors are grateful to Dr. A. Ya. Rusanov for the enlightening discussions and correspondence.

References

1. H. Feldmeier, Rep. Prog. Phys. **50**, 915 (1987).
2. J.R. Nix, A.J. Sierk, in *Proceedings of the International School Seminar on Heavy Ion Physics, Dubna, USSR, 1986*, edited by M.I. Zarubina, E.V. Ivashkevich (JINR, Dubna, 1987) p. 453; in *Proceedings of the Sixth Adriatic Conference on Nuclear Physics: Frontiers of Heavy Ion Physics, Dubrovnik, Yugoslavia, 1987*, edited by N. Cindro R. Caplar, W. Greiner (World Scientific, Singapore, 1990) p. 333.
3. D. Hilscher, H. Rossner, Ann. Phys. (Paris) **17**, 471 (1992).
4. D. Hilscher, I.I. Gontchar, H. Rossner, Yad. Fiz. **57**, 1255 (1994).
5. F. Gönnenwein, in *Nuclear Fission Process*, edited by C. Wagemans (CRC Press, Boca Raton, FL, 1991) p. 287.
6. P. Fröbrich, H. Rossner, Z. Phys. A **349**, 99 (1994).
7. V.A. Drozdov, D.O. Eremenko, S. Yu. Platonov, O.V. Fotina, O.A. Yuminov, Yad. Fiz. **64**, 221 (2001) (Phys. At. Nucl. **64**, 179 (2001)).
8. A.V. Karpov, P.N. Nadtochy, D.V. Vanin, G.D. Adeev, Phys. Rev. C **63**, 054610 (2001).
9. M. Brack, J. Damgaard, A.S. Jensen, H.C. Pauli, V.M. Strutinsky, C.Y. Wong, Rev. Mod. Phys. **44**, 320 (1972).
10. H.A. Kramers, Physica (Utrecht) **7**, 284 (1940).
11. A.V. Ignatyuk, M.G. Itkis, V.N. Okolovich, G.N. Smirenkin, A.S. Tishin, Yad. Fiz. **21**, 1185 (1975) (Sov. J. Nucl. Phys. **21**, 612 (1975)).
12. L.D. Landau, E.M. Lifshitz, *Statistical Physics Part 1* (Pergamon Press, Oxford, 1980).
13. P. Fröbrich, I.I. Gontchar, Phys. Rep. **292**, 131 (1998).
14. P.N. Nadtochy, A.V. Karpov, G.D. Adeev, Yad. Fiz. **65**, 832 (2002) (Phys. At. Nucl. **65**, 799 (2002)).
15. A.S. Iljinov, M.V. Kazarnovsky, E. Ya. Paryev, *Intermediate Energy Nuclear Physics* (CRC, Boca Raton, 1994); Ye. S. Golubeva, A.S. Iljinov, A.S. Botvina, N.M. Sobolevsky, Nucl. Phys. A **483**, 539 (1988).
16. W.D. Myers, W.J. Swiatecki, Ark. Fys. **36**, 343 (1967).

17. G.D. Adeev, L.A. Filipenko, P.A. Cherdantsev, Yad. Fiz. **23**, 30 (1976) (Sov. J. Nucl. Phys. **23**, 15 (1976)); R.K. Gupta, W. Scheid, W. Greiner, Phys. Rev. Lett. **35**, 353 (1975).
18. G.D. Adeev, I.I. Gonchar, L.A. Marchenko, Yad. Fiz. **42**, 42 (1985) (Sov. J. Nucl. Phys. **42**, 25 (1985)).
19. W. Greiner, M. Ivascu, D. Poenaru, A. Sandulescu, *Treatise on Heavy Ion Science*, edited by D.A. Bromley, Vol. **8** (Plenum, New York, 1988) p. 641.
20. G.D. Adeev, Fiz. Elem. Chastits At. Yadra **23**, 1572 (1992) (Sov. J. Part. Nucl. **23**, 684 (1992)).
21. E.S. Hernandez, W.D. Myers, J. Randrup, B. Remaud, Nucl. Phys. A **361**, 483 (1981).
22. H. Freiesleben, J.V. Kratz, Phys. Rep. **106**, 1 (1984).
23. K.T.R. Davies, A.J. Sierk, J.R. Nix, Phys. Rev. C **13**, 2385 (1976).
24. B. Martschew, K. Pomorski, Acta Phys. Pol. B **13**, 747 (1982).
25. L.D. Landau, E.M. Lifshitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1987).
26. J. Randrup, W.J. Swiatecki, Nucl. Phys. A **429**, 105 (1984).
27. U. Brosa, H.J. Krappe, Z. Phys. A **287**, 65 (1978); U. Brosa, D.H.E. Gross, Z. Phys. A **294**, 217 (1980).
28. R.W. Hasse, P. Nerud, J. Phys. G **2**, L101 (1976).
29. R. Wieczorec, R.W. Hasse, G. Süssmann, in *Proceedings of the Third Symposium on the Physics and Chemistry of Fission, Rochester, 1973*, Vol. **1** (IAEA, Vienna, 1974) p. 523.
30. W.D. Myers, W.J. Swiatecki, T. Kodama, L.J. Al-Jaick, E.R. Hilf, Phys. Rev. C **15**, 2032 (1977).
31. B. Bush, Y. Alhassid, Nucl. Phys. A **531**, 27 (1991).
32. A.J. Sierk, J.R. Nix, Phys. Rev. C **21**, 982 (1980).
33. J. Blocki, Y. Boneh, J.R. Nix, J. Randrup, M. Robel, A.J. Sierk, W.J. Swiatecki, Ann. Phys. (N.Y.) **113**, 330 (1978).
34. H. Hofmann, F.A. Ivanyuk, C. Rummel, S. Yamaji, Phys. Rev. C **64**, 054316 (2001).
35. B.W. Bush, G.F. Bertsch, B.A. Brown, Phys. Rev. C **45**, 1709 (1992).
36. S. Yamaji, H. Hofmann, R. Samhammer, Nucl. Phys. A **475**, 487 (1988); F.A. Ivanuyk, H. Hofmann, Nucl. Phys. A **657**, 19 (1999).
37. H.G. Clerc, W. Lang, H. Wohlfarth, H. Schrader, K.H. Schmidt, in *Proceedings of the Forth Symposium on the Physics and Chemistry of Fission, Jülich, 1979*, Vol. **2** (IAEA, Vienna) p. 65; W. Lang, H.G. Clerc, H. Wohlfarth, H. Schrader, K.H. Schmidt, Nucl. Phys. A **345**, 34 (1980).
38. G.D. Adeev, I.I. Gontchar, V.V. Pashkevich, O.I. Serdyuk, Yad. Fiz. **50**, 1242 (1989) (Sov. J. Part. Nucl. **50**, 774 (1989)).
39. H. Hofmann, A.S. Jensen, F. Scheuter, in *Proceedings of the 12th International Symposium on Nuclear Physics, Gaussing, GDR, 1982*, Report NISSN 0138-2950, p. 96.
40. S. Ayik, E. Suraud, J. Stryjewski, M. Belkacem, Z. Phys. A **337**, 413 (1990); Y. Abe, S. Ayik, P.-G. Reinhard, E. Suraud, Phys. Rep. **275**, 49 (1996).
41. J.A. McHugh, M.C. Michel, Phys. Rev. **172**, 1160 (1968).
42. K.T.R. Davies, R.A. Managan, J.R. Nix, A.J. Sierk, Phys. Rev. C **16**, 1890 (1977).
43. G.D. Adeev, I.I. Gontchar, V.V. Pashkevich, N.I. Pischasov, O.I. Serdyuk, Fiz. Elem. Chastits At. Yadra **19**, 1229 (1988) (Sov. J. Part. Nucl. **19**, 529 (1988)).
44. G.I. Kosenko, I.G. Cagliari, G.D. Adeev, Yad. Fiz. **60**, 404 (1997) (Phys. At. Nucl. **60**, 334 (1997)); G.I. Kosenko, D.V. Vanin, G.D. Adeev, Yad. Fiz. **61**, 2142 (1998) (Phys. At. Nucl. **61**, 2031 (1998)).